4.2 Answers

Answer 1

(a)

$$\begin{aligned} g(x) &= 0 \\ e^{x-1} + x - 6 &= 0 \\ e^{x-1} &= 6 - x \\ ln(e^{x-1}) &= ln(6 - x) \\ x - 1 &= ln(6 - x) \\ x &= ln(6 - x) + 1 \qquad \square \end{aligned}$$

(b) $x_0 = 2$
$x_1 = 2.3863$
$x_2 = 2.2847$
$x_3 = 2.3125$

(c) g (2.30**65**) = - 0.000 27
g (2.30**75**) = + 0.004 4
One answer is positive and one is negative.
As the function is smooth and continuous there must be a value of x between 2.3065 and 2.3075 that causes $g(x) = 0$.
Furthermore, all numbers in this interval round to 2.307 and so the root must be 2.307 to three decimal places.

Answer 2

(a) $x_0 = 2.5$
$x_1 = 2.32$
$x_2 = 2.371\,581\,451$
$x_3 = 2.355593575$
$x_4 = 2.360436923$

(b) Let $g(x) = -x^3 + 2x^2 + 2$
g (2.35**85**) = + 0.005 835 77
g (2.35**95**) = - 0.001 422 86
One answer is positive and one is negative.
As the function is smooth and continuous there must be a value of x between 2.3585 and 2.3595 that causes $g(x) = 0$.
Furthermore, all numbers in this interval round to 2.359 and so the root must be 2.359 to three decimal places.

Answer 3

(a)

$$\begin{aligned} f(x) &= 0 \\ x^3 + 2x^2 - 3x - 11 &= 0 \\ x^3 + 2x^2 &= 3x + 11 \\ x^2 (x + 2) &= 3x + 11 \\ x^2 &= \frac{3x + 11}{x + 2} \qquad x \neq 2 \\ x &= \sqrt{\left(\frac{3x + 11}{x + 2} \right)} \qquad x \neq 2 \qquad \square \end{aligned}$$

(b) $x_1 = 0$

$x_2 = 2.345\ 207\ 88$

$x_3 = 2.037\ 324\ 945$

$x_4 = 2.058\ 748\ 112$

(c) $f(\,2.05\underline{\mathbf{65}}\,) = -\ 0.013\ 781\ 637$

$f(\,2.05\underline{\mathbf{75}}\,) = +\ 0.004\ 140\ 109\ 4$

One answer is positive and one is negative.

As the function is smooth and continuous there must be a value of x between 2.0565 and 2.0575 that causes $f(x) = 0$. Furthermore, all numbers in this interval round to 2.057 and so the root must be 2.057 to three decimal places.

Answer 4

(a) $f\,(\,1.2\,) = +\,0.491\;665\;51$

$f\,(\,1.3\,) = -\,0.048\;719\;817$

One answer is positive and one is negative.

As the function is smooth and continuous there must be a value of x between 1.2 and 1.3 that causes $f\,(x) = 0$.

(b)

$$
\begin{aligned}
f\,(x) &= 0 \\
4\,cosec\,x - 4x + 1 &= 0 \\
\frac{4}{sin\,x} - 4x + 1 &= 0 \\
\frac{1}{sin\,x} - x + \frac{1}{4} &= 0 \\
x &= \frac{1}{sin\,x} + \frac{1}{4} \qquad sin\,x \neq 0 \qquad \square
\end{aligned}
$$

(c) $x_0 = 1.25$

$x_1 = 1.3038$

$x_2 = 1.2867$

$x_3 = 1.2917$

(d) $f\,(\,1.29\underline{\mathbf{05}}\,) = +\,0.000\;445\;666\;95$

$f\,(\,1.29\underline{\mathbf{15}}\,) = -\,0.004\;750\;172\;79$

One answer is positive and one is negative.

As the function is smooth and continuous there must be a value of x between 1.2905 and 1.2915 that causes $f\,(x) = 0$.

Furthermore, all numbers in this interval round to 1.291 and so the root must be 1.291 to three decimal places.